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PureCake

A Verified Compiler for a Lazy Functional Language

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Language guarantees

Implementing MyCriticalSoftware

C	ML	
?	type safety	type safety
	memory safety	memory safety
		purity vs. I/O
		ref. transparency
		laziness
		free theorems

Compiler guarantees

Compiling MyCriticalSoftware

C

CompCert

`0xa1b2c3...`

✓

ML

CakeML

`0xbcd456...`

✓

PureCake

`0xf9e8d7...`

✓

Why verified compilation?

How do we know what our programs will do?

understand source semantics + **trust compiler to preserve it**

We may trust GHC — but does its source semantics match our understanding?

Verified compilation gives us **a verified link** between:

- formally-specified source semantics
- semantics-preserving compilation

Our work

The **PureCake** project:

a HOL4-**verified compiler** for

a **lazy**, **purely functional** language which

is inspired by **Haskell** and

targets **CakeML**

CakeML = a verified implementation of a subset of ML [POPL14]

Key contributions

Highlights

- sound equational reasoning
- parsing expression grammar (PEG) for Haskell-like syntax
- two-phase constraint-based type inference*
- demand analysis*
- optimisations for non-strict idioms
- monadic reflection* (monadic → imperative)
- CakeML as a back end for end-to-end verified compilation

***Not mechanically verified before**

This talk

High level, whistle-stop tour!

For more details:

- Read our paper: cakeml.org/pldi23-purecake.pdf
- Visit our GitHub: github.com/cakeml/pure
- Talk to us!

Roadmap

Introduction

Source language

Compiler front end

Compiler back end

Connection with CakeML

A realistic functional language

PureLang has standard functional idioms ...

```
fact :: Integer -> Integer -> Integer
fact a n =
  if n < 2 then a
  else fact (a * n) (n - 1)
```

general recursion

```
map :: (a -> b) -> [a] -> [b]
map f l = case l of
          [] -> []
          h:t -> f h : map f t
```

algebraic data types + pattern-matching

```
factorials :: [Integer]
factorials = map (fact 1) (numbers 0)
```

higher-order functions

A realistic subset of Haskell

… and Haskell extras

```
numbers :: Integer -> [Integer]
numbers n = n : numbers (n + 1)


main :: IO ()
main = do
  n <- readInt -- read from stdin
  let facts = take n factorials
  app (\i -> print $ toString i) facts
```

laziness → infinite data

pure by default, monads for:

- sequencing
- stateful computations
- I/O

Single `IO` monad for arrays, exceptions, and I/O (via FFI calls)

Also: indentation-sensitivity, `do` notation, mutual recursion, …

Formal syntax

A tale of two ASTs… separate implementation and verification

$$ce \xrightarrow{\text{desugar}} e$$

compiler *expressions*

- higher-level
- used in implementation
- includes `case`

semantic *expressions*

- ground truth for semantics
- constructor operations: test name/arity equality & argument projection

Semantics — ITrees

Need to model non-termination and I/O

HOL4-expressible coinductive *interaction trees [Xia et. al., POPL20]*:

$$\begin{array}{rcll} \mathsf{itree}\ E\ A\ R & ::= & \mathsf{Ret}\ R & \textit{termination} \\ & \mid & \mathsf{Vis}\ E\ (A \to \mathsf{itree}\ E\ A\ R) & \textit{I/O via FFI channel} \\ & \mid & \mathsf{Div} & \textit{silent divergence} \end{array}$$

vs.

No Tau nodes? Div!?

rely on non-constructivity of HOL4's logic

strong bisimulation coincides with equality

Semantics — definitions

Operational semantics in layers:

1. Weak-head evaluation: call-by-name, functional big-step

$$\mathrm{eval}^{n}_{\mathrm{wh}}\ e = wh$$

2. Lift to unclocked evaluation

$$\mathrm{eval}_{\mathrm{wh}}\ e \overset{\mathrm{def}}{=} \begin{cases} wh & \text{for } \textbf{some } n,\ \mathrm{eval}^{n}_{\mathrm{wh}}\ e = wh \neq \mathrm{Timeout} \\ \mathrm{Timeout} & \text{for } \textbf{all } n,\ \mathrm{eval}^{n}_{\mathrm{wh}}\ e = \mathrm{Timeout} \end{cases}$$

3. Stateful interpretation of `IO` operations

$$(\!|\, -,\ -,\ - \,|\!) : wh \to \kappa \to \sigma \to \mathrm{itree}\ E\ A\ R$$

Finally, $[\![\, e \,]\!] \overset{\mathrm{def}}{=} (\!|\, \mathrm{eval}_{\mathrm{wh}}\ e,\ \varepsilon,\ \varnothing \,|\!)$

Mechanised equational reasoning

Mechanise untyped applicative bisimulation, $\cong$ *[Abramsky, 1990]*

Proved *congruent* via Howe's method *[Howe, 1996]*
i.e. bisimilar sub-expressions $\implies$ *bisimilarity*

Definitions:

- α-equivalence: $e_1 =_\alpha e_2 \overset{\text{def}}{=} \text{perm\_vars}\ e_1\ e_2$
- β-equivalence: $(\lambda x.\, e_1) \cdot e_2 =_\beta (\text{freshen}_{e_2}\ e_1)[{}^{e_2}/_{x}]$
- A standard contextual equivalence: $e_1 \sim e_2$
  (equality under all closing contexts)

Derived results:

$$e_1 \cong e_2 \iff e_1 \sim e_2 \qquad \frac{e_1 =_\alpha e_2}{e_1 \cong e_2} \qquad \frac{e_1 =_\beta e_2}{e_1 \cong e_2}$$

Type system

Standard Hindley-Milner rules… with an unusual soundness proof

Problem:

non-strict semantics + exhaustive `case` splits mean that
“preservation” (subject reduction) does not hold

Solution:

- Define an alternative syntax for typing
- Prove subject reduction by construction
- Use equational theory to bridge the gap to original syntax

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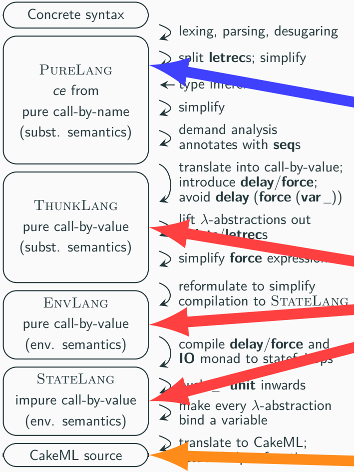
Compiler structure



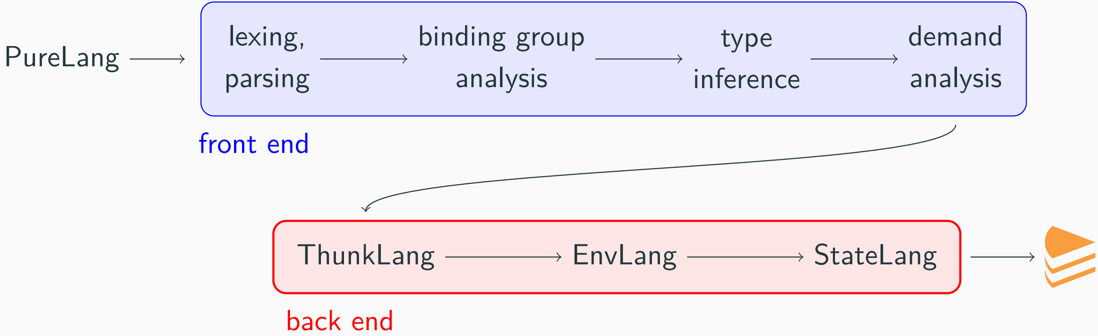
Upcoming slides examine the compiler top to bottom

Frontend accepts PureLang

Three intermediate languages, each with a specific focus

Targets CakeML

Compiler structure



PureLang → lexing, parsing → binding group analysis → type inference → demand analysis

front end

ThunkLang → EnvLang → StateLang →

back end

Parsing

Indentation-sensitive parsing expression grammar (PEG):

PEG + *[Adams POPL13]*

$$N \to X_1^{\mathcal{R}_1}\, X_2^{\mathcal{R}_2}\, \dots\, X_n^{\mathcal{R}_n}$$

where $\mathcal{R} \in \{ =,\ >,\ \geq,\ \mathcal{U} \}$

- Symbolic sets of possible indentations for each non-terminal
- Verified to terminate on all inputs

Binding group analysis

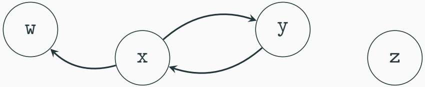
Parsing

```
z = 42
y = x + 1
x = w + y
w = 0
main
```

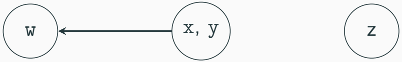
⟶

```
let rec z = 42
        y = x + 1
        x = w + y
        w = 0
    in main
```

Analyse dependencies



Pseudo-topological sort



Transform code + tidy

```
let w = 0 in
let rec x = w + y ; y = x + 1
    in main
```

Verified entirely within equational theory

Sound constraint-based type inference

Two-phases: generate *all* constraints ⟶ solve constraints

Subset of Helium's $\textsc{Top}$ framework *[Heeren et. al., Haskell 2003]*

- Open to high-quality error messages
- Path to various Haskell 98 features

Soundness theorem:

$$\begin{aligned} &\text{infer } \mathit{ce} \ \ \textbf{succeeds} \\ &\qquad \Longrightarrow\ \mathit{ce} \vdash_{\textsc{Top}} \text{cs} \quad \textit{and} \quad \text{cs } \textbf{solveable} \\ &\qquad \Longrightarrow\ \Gamma \vdash \mathit{ce} : \tau \end{aligned}$$

Demand analysis

Avoid excessive thunks — `acc` heap-allocated each recursive call!

```
fact acc n =
     if ... then acc else fact (acc * n) (n - 1)
```

- e **demands** $\overline{x_n} \overset{\text{def}}{=} e \cong (x_1$ `` `seq` `` $\ldots$ x_n `` `seq` `` $e)$
- Implement/verify* analysis: e **demands** (analyse e)
- Prefix code with `seq`, including in recursive functions

*Verify with an alternative equational theory, $\approx$ *[Sergey et. al., 2014]*:

stuck $\approx$ diverged $\approx \bot$ **but** well-typed $\Longrightarrow$ $\approx$, $\cong$ coincide

`seq`-prefixing preserves typing

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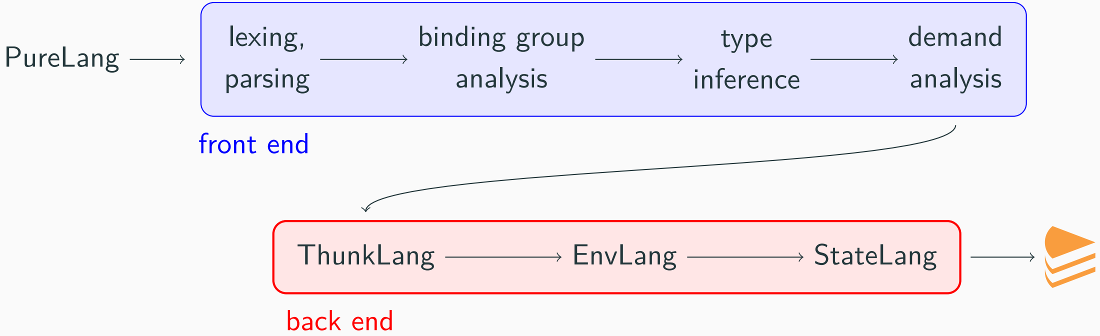

PureLang → lexing, parsing → binding group analysis → type inference → demand analysis

front end

ThunkLang → EnvLang → StateLang →

back end

Methodology — implementation vs. verification

Prior work: (such as CakeML)

- Define implementation function: $\text{transform} : e \to e$
- Verify: $\text{wf } e \implies [\![\, \text{transform } e \,]\!] = [\![\, e \,]\!]$

This work:

$$e \;\mathcal{R}\; e'$$

syntactic relations

$$\text{compile } ce = ce'$$

code transformation

- for **verification**
- an implementation *envelope*

- for **implementation**
- must fit in relation envelope

Methodology — workflow

1. **Define** and **verify** $\mathcal{R}$: $\quad e \; \mathcal{R} \; e' \implies [\![e]\!] = [\![e']\!]$

   Three simulation proofs: one per layer of the semantics
2. **Define** $\text{compile} : \mathit{ce} \to \mathit{ce}$
3. **Verify** $\text{wf } \mathit{ce} \implies (\text{desugar } \mathit{ce}) \;\; \mathcal{R} \;\; (\text{desugar}\,(\text{compile } \mathit{ce}))$
4. **Compose** theorems:

   $$\text{wf } \mathit{ce} \implies [\![\text{desugar } \mathit{ce}]\!] = [\![\text{desugar}\,(\text{compile } \mathit{ce})]\!]$$
5. **Integrate** into compiler, discharge wf *ce*

Separation of concerns for modularity and ease-of-verification

ThunkLang

Call-by-value semantics

Syntax: $e \ ::= \ \ldots \mid \textbf{delay}\ e \mid \textbf{force}\ e$ introduce *thunk*s

Semantics:

$$\text{eval}\ (\textbf{delay}\ e) = \textbf{thunk}\ e \qquad \frac{\begin{array}{c}\text{eval}\ e = \textbf{thunk}\ e' \\ \text{eval}\ e' = v\end{array}}{\text{eval}\ (\textbf{force}\ e) = v}$$

NB thunks are pure, value-sharing comes later

Optimisation: reduce **delay** (**force** x); two forms of restricted CSE

Verification: *seven* syntactic relations in total

EnvLang

Environment-based semantics + minor reformulations

Syntax:	essentially unchanged
Semantics:	~~substitutions~~ **environments**
Verification:	focuses on the change in semantic style

StateLang

IO monad compiled to **effectful primitives, thunks shared statefully**

Syntax: $e \ ::= \ \ldots \mid \texttt{malloc}\ n\ e \mid \ldots$

remove **delay**/**force**, **return**/**bind**/...

Semantics: *stateful* CESK machine

Compilation:

$$\textbf{alloc}\ n\ e \longmapsto \lambda\_.\ \texttt{malloc}\ n\ e$$

$$\textbf{force}\ e \longmapsto \begin{array}{l} \textbf{let}\ x = e'\ \textbf{in} \\ \textbf{if}\ x[0]\ \textbf{then}\ x[1] \\ \textbf{else}\ \ldots\ x[0] := \textbf{true};\ x[1] := v\ \ldots \end{array}$$

$[\textbf{true},\ v]$ *or* $[\textbf{false},\ \lambda\_.\ \ldots]$

Optimisation: simplify $\lambda\_.\ e$ and **unit**

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Oracles vs. ITrees

Reconciling differing semantic styles

$$o_1 \xrightarrow{\Delta(o_1)} o_2 \xrightarrow{\Delta(o_2)} \dots$$

linear oracles: $\text{semantics}_{\Delta}\ e = tr$

$$\begin{tikzcd}
 & \text{Vis } o_1\ k_1\ \dots \\
\text{Vis } o\ k \arrow[ur, "k(a_1)"] \arrow[r] & \text{Vis } o_2\ k_2\ \dots \\
 & \vdots
\end{tikzcd}$$

branching ITrees: $[\![\, e \,]\!] = \text{Vis } \dots$

- Verified ITree semantics: $[\![\, e \,]\!]_{\text{🍰}} \overset{\Delta}{\rightsquigarrow} tr \iff \text{semantics}_{\Delta}\ e = tr$
- New compiler correctness:

$$\frac{\begin{array}{c}\text{cakeml } e = \text{Some } code \\ code\ \textbf{in memory of}\ machine\end{array}}{[\![\, machine \,]\!]_{\mathrm{M}}\ \text{prunes}\ [\![\, e \,]\!]_{\text{🍰}}}$$

Compiler correctness

$$\frac{\text{purecake } \mathit{str} = \text{Some } \mathit{ast}_{🍰}}{\begin{array}{c}\textbf{exists } \mathit{ce} \textbf{ such that} \\ \text{frontend } \mathit{str} = \text{Some } (\mathit{ce}, \_) \\ \mathit{ce} \textbf{ is type safe} \\ [\![\, \text{desugar } \mathit{ce} \,]\!]_{\text{pure}} \simeq [\![\, \mathit{ast}_{🍰} \,]\!]_{🍰}\end{array}}$$

End-to-end correctness

$$\frac{\begin{array}{c}\text{purecake } \mathit{str} = \text{Some } \mathit{ast}_{\text{🍰}} \\[1em] \text{cakeml } \mathit{ast}_{\text{🍰}} = \text{Some } \mathit{code} \\[1em] \mathit{code}\ \textbf{in memory of}\ \mathit{machine}\end{array}}{\begin{array}{c}\textbf{exists}\ \mathit{ce}\ \textbf{such that} \\[1em] \text{frontend } \mathit{str} = \text{Some } (\mathit{ce},\ \_) \\[1em] [\![\, \mathit{machine} \,]\!]_{\text{M}}\ \text{ prunes }\ [\![\, \text{desugar } \mathit{ce} \,]\!]_{\text{pure}}\end{array}}$$

A verified binary

Verified bootstrapping using *proof-producing synthesis [ICFP12]*

$$\text{HOL4 functions} \xrightarrow{\text{synthesise}} \mathit{AST}_{🍰} + \mathit{equivalence\ proof}$$

The PureCake compiler is a HOL4 function...

$$\text{purecake} \xrightarrow{\text{synthesise}} \mathit{purecake\_AST}_{🍰} + \mathit{equivalence\ proof}$$

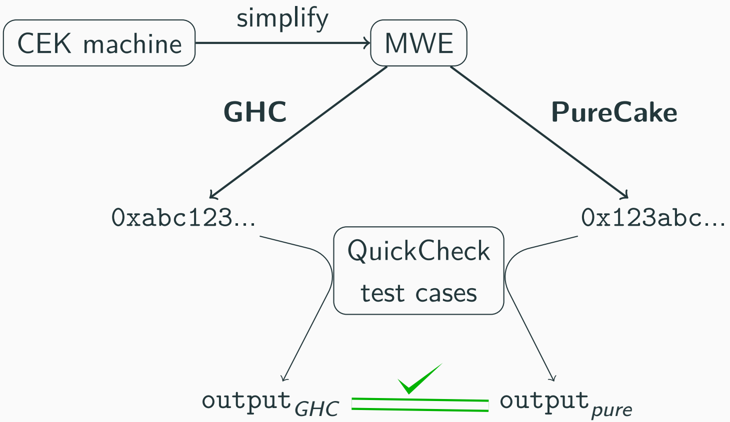
... and the CakeML compiler can be evaluated in-logic:

$$\vdash \text{cakeml}\ (\mathit{purecake\_AST}_{🍰}) = \texttt{0xabc123...}$$

Equivalence proofs transport verification to the binary

Real-world usage of PureCake

Testing on the Cardano block chain platform by QuviQ



Summary

PureCake

cakeml.org/purecake

a **verified compiler** for a Haskell-like language

- sound equational reasoning
- Haskell-like syntax
- two-phase constraint-based sound type inference
- verified demand analysis
- optimisations to handle non-strict code realistically
- end-to-end guarantees by targeting CakeML
- feasible to use on real code

Questions?

Backup slides

Future work

Only a first version! Many possible extensions, for example:

- Increasing source **expressivity** (e.g. for `case`)
- More Haskell 98 types, e.g. **typeclasses**
- More effective **demand analysis**
- Back end **optimisations**

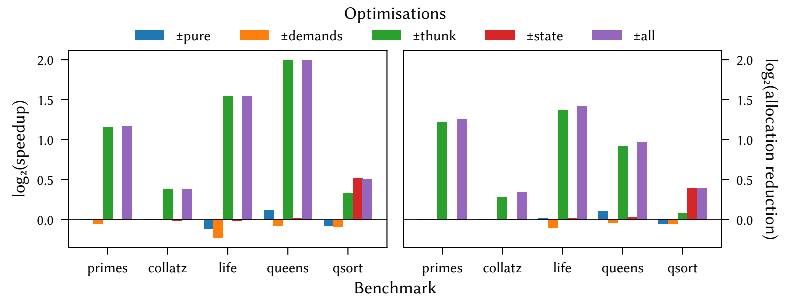
A **verified REPL** for PureCake *[Sewell et. al., PLDI23]*

Evaluation — setup

Measure **execution time** and **memory allocations**

- Turn off individual optimisations to highlight their effect
  - pure: binding group analysis
  - demands: demand analysis
  - thunk: some **force** (**delay** *e*) reduction and CSE in ThunkLang
  - state: $\lambda\_.\ e/$**unit** optimisations in StateLang
- Five benchmarks, each accepting integer *n* input
  - primes: *n*th prime calculation
  - collatz: longest Collatz sequence for a number less than *n*
  - life: Conway's Game of Life for *n* generations
  - queens: solutions for the *n*-queens problem
  - qsort: imperative quicksort for an array of length *n*

Evaluation — results



Results

- ThunkLang optimisations provide significant benefit
- StateLang optimisations improve monadic code particularly
- Binding group analysis negligible
- Regressions for demand analysis: `seq`-insertion is too eager!

Alternative compiler correctness

$$\frac{\text{frontend}\ \mathit{str} = \text{Some}\ (\mathit{ce},\ \_)}{\begin{array}{c} \mathit{ce}\ \textbf{is type safe} \\ \textbf{exists}\ \mathit{ast}_{\text{🍰}}\ \textbf{such that} \\ \text{purecake}\ \mathit{str} = \text{Some}\ \mathit{ast}_{\text{🍰}} \\ [\![\, \text{desugar}\ \mathit{ce} \,]\!]_{\text{pure}} \approx [\![\, \mathit{ast}_{\text{🍰}} \,]\!]_{\text{🍰}} \end{array}}$$